Film Formation in Two-Phase Annular Flow

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The liquid film in annular two-phase flow is maintained by droplet movement to and from the gas core and by wave action. This work examines droplet movement and its effect on the liquid film. Film formation and movement both axially and circumferentially were investigated by using transient techniques.

The simultaneous flow of a gas and a liquid in a closed conduit occurs frequently in engineering practice. If the conduit is a horizontal pipeline and the phases are flowing concurrently, there are seven visually classified flow patterns that can exist (*2*), depending on the mass flow rates of each phase, the physical properties of each phase, and the pipeline geometry. For high mass flow rates of gas (for example, 10,000 lb./hr. × sq.ft. for air) and a ratio of the liquid mass flow rate to the gas mass flow rate of 100 or less (*4*), the flow pattern that will exist is called *annular flow*.

In fully developed annular flow, the gas flows as a high speed core down the center of the pipeline. The liquid flows in two ways: in an annular film around the pipe wall and as droplets entrained by the gas core. For a given liquid rate, the fraction of the liquid flowing as entrained droplets increases as the gas rate is increased. There is a dynamic equilibrium between the droplets in the gas core and the film. New droplets are continually being formed at the gas-liquid interface, and droplets in the core continually impinge on the film. This phenomenon is known as *interchange*.

The effect of these entrained droplets on heat and mass transfer in annular flow is of obvious importance, since the droplets greatly increase the interfacial area between the two phases. Moreover, because of interchange, new droplets are continually being generated at the gas-liquid interface, while droplets which have been in the gas core return to the film. This convected transport will greatly affect the overall rates of mass, momentum, and heat transfer.

In order to quantitatively predict the effect of interchange, it would be necessary to have the following information:

1. The rate of droplet generation and its circumferential variation.
2. Information on the movement of droplets while they are in the gas core.
3. The rate of droplet deposition and its circumferential variation.

This paper presents experimental results from a 1 in. I.D. horizontal pipeline on the rate of droplet deposition and its circumferential variation. Two kinds of interchange from the gas core to the liquid film were measured. One is a value of interchange which is an average value for the entire pipe circumference. In fully developed annular flow, this will also be the circumferential averaged value of interchange from the film to the core because of the dynamic equilibrium which exists. In the paper, this kind of interchange is denoted by I and has units of pounds per second per foot of pipe. The second kind of interchange measured is a value for a specified circumferential position; it is denoted by G and has units of pounds per second per square foot of inside pipe surface.

The experimental technique that was developed to measure the interchange to the film consisted of reducing the film thickness so that droplet generation ceased and then measuring the film flow rate at several axial positions downstream from the point of film thickness reduction. Between any two axial positions, the circumferential averaged value of the interchange to the film can be calculated from the difference of the film flow rates. Because there is not any droplet generation after reduction of the film thickness, the amount of liquid flowing as droplets decreases owing to this interchange. The circumferential averaged value of interchange obtained between two downstream axial positions is not the equilibrium value for the existing flow conditions. A mathematical model is proposed to obtain this equilibrium value from the nonequilibrium experimental data.

Throughout the paper, the concept of an equilibrium and a nonequilibrium value of a variable will be used. The experiment is a transient one, but the transient variable is not time. It is the downstream axial distance from the film thinner. The equilibrium value is the value for fully developed annular flow and also the value of the variable at the origin of the axial coordinate system. A nonequilibrium value of a variable is a value which pertains to some axial position downstream of the film thinner. If it is not specifically mentioned that a value of a variable is the equilibrium or the nonequilibrium value, it is understood to be the equilibrium value.

To obtain the circumferential dependence of the interchange to the film, a partial differential equation in axial position and circumferential position is obtained from a material balance on a differential segment of film. Numerical values for interchange as a function of axial and circumferential position are calculated by numerical differentiation of experimental data on film thickness, axial velocity, and circumferential velocity. These nonequilibrium values of interchange are extrapolated to find the corresponding equilibrium values for each of ten circumferential positions.

PREVIOUS RESULTS

The techniques that have been used to measure interchange can be divided into four groups. One technique is to approximate the annular flow situation by a turbulent gas flow in a conduit. A water spray is injected into the gas stream, and the rate of liquid transport to the wall of the conduit is measured. Another technique is to use a tracer stream which allows measurement in fully developed annular flow. The other two techniques are transient; one consists of introducing a film onto the inside wall of a pipe in which gas is flowing, while the other consists of removing the film from fully developed annular flow and measuring the film buildup.

Alexander and Coldren (*1*) and Paleev and Filippovich (*8*) measured the rate of transport of water drops from a turbulent air stream to the wall of a horizontal conduit.

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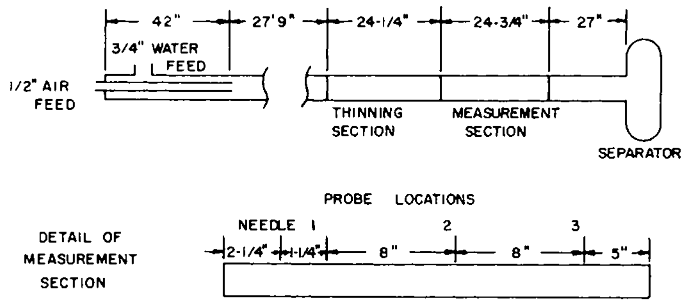

Fig. 1. Schematic drawing of pipeline.

Alexander and Coldren used a 1.86 in. I.D. pipe, while Paleev and Filippovich used a 4 × 40 mm. rectangular cross-section channel. Both these studies used a drop size distribution that was not the same as the drop size distribution that would be in fully developed annular flow, since the drops were injected into the gas stream as opposed to being generated as the gas-liquid interface.

Quandt (9) measured interchange in a vertical rectangular cross-section conduit. The experimental technique involved the measurement of the response to a step injection of a dye stream into the film. The dye concentration of the film was measured at several axial positions downstream of the injection point. From the linear plot of the logarithm of a function of film dye concentration against the axial distance downstream of the injection point, it was possible to obtain an experimental value of the entrainment from the ordinate intercept of the line. By using this value of entrainment and the slope of the line, an experimental value of interchange was obtained.

Cousins et al. (5) used the same dye injection technique as Quandt to measure interchange in a ⅜ in. I.D. vertical pipe. They improved the technique, however, by obtaining independent values of entrainment.

By using ammonia as a tracer stream, Anderson, Bollinger, and Lamb (3) measured the circumferential averaged value of interchange in a 1 in. horizontal pipe from the results of mass transfer experiments. Two critical assumptions in the analysis are that the entrained drops and the gas core are in equilibrium and that the gas-film mass transfer coefficient can be predicted from the extrapolation of the j factor correlation for wetted wall columns. There is not any way to verify the second assumption, but some calculations that were done show that the first assumption is probably incorrect. Because of the above mentioned difficulties, this method is not an ideal one with which to measure interchange.

To use a dye injection technique to measure interchange in horizontal annular flow would be difficult. The difficulty would be the necessity of having a two-dimensional model, since circumferential flow is important in horizontal flow. To analyze this two-dimensional model, accurate data would have to be obtained on the circumferential flow, and to date there has not been any satisfactory experimental technique developed which can be used to measure this flow.

Hoogendoorn and Welling (7) measured the circumferential averaged value of interchange from the film to the core for horizontal annular flow. An annular film was formed by injecting liquid through a porous section into a pipe in which air was already flowing. The amount of liquid which entered the gas core in a given length of pipe as drops was calculated from the difference between the flow rate of liquid through the porous section and the film flow rate downstream of the injection point. The analysis did not take into account the entrance effects in which the film attains its asymmetry and the roll waves grow. The average film thickness in horizontal annular flow is about five times greater in the bottom of the pipe than it is in the top of the pipe. This asymmetry is caused by the circumferential flow of the film due to gravity. Woodmansee and Hanratty (*14*) have found that the drops are generated from the crest of the roll waves which exist in annular flow, so that until the roll waves are formed, there would not be any droplet generation. A certain entrance distance would be required for the circumferential film flow to affect the asymmetry and for the roll waves to grow.

Cousins et al. (5) used another technique besides dye injection to measure interchange. This consisted of removing the film measuring the film buildup downstream of the film removing position. The present experiment was suggested from this technique.

EXPERIMENTAL WORK

The experiments were done in a 1 in. I.D. horizontal pipeline by using air and water that were filtered before they were introduced into the test section. The water flow rate was measured by using a rotometer, while the air flow rate was measured with a sharp edged orifice. Figure 1 is a schematic diagram of the pipeline showing the details of the entrance section and the arrangement of the film thinner and various measuring devices.

The purpose of the film thinner was to reduce the film thickness to a small enough value so that the rate of a droplet generation was negligible. This method of measuring interchange was suggested by the work of Cousins et al. (5). A sketch of the device used to reduce the film thickness appears in Figure 2. The methyl methacrylate sleeve is divided into two sections since, owing to asymmetry of the annular film, it was necessary to remove more liquid at the bottom of the pipe than at the top.

The assumption that there was negligible droplet generation in the measurement section was verified in two ways. The maximum value of the film thickness downstream of the film thinner was less than the minimum value required for film atomization according to the criteria of van Rossum (*12*). A dye stream was injected into the film at the bottom of the pipe at a point just downstream of the film thinner. The dye stream remained in the bottom of the pipe, and did not appear in the film at any other circumferential position. In fully developed annular flow, a dye stream similarly injected would be dispersed throughout the entire film by the interchange processes.

The film thickness was measured by using a technique developed by Swanson (*11*). A probe was built by placing the parallel wires perpendicular to the direction of flow and by stretching these wires across the diameter of the pipe. An electrical circuit is formed by the wires and the liquid film between them, the impedance of which is proportional to the length of the wires covered with liquid. By insulating one end of these wires, it was possible to measure the film thickness at only one side of the pipe.

The axial velocity of the film was determined from the time required for a pulse of salt to travel between two points in the pipeline. The concentration of salt in the liquid film was measured with a conductivity probe technique developed by Russell (*10*).

The circumferential velocity was determined by measuring the deflection of a tracer stream. Several different tracers were used, but they all had the common characteristic that their deflection was visually measured. From the measured angular deflection of the tracer stream and the axial velocity, it was possible to compute the circumferential velocity.

The axial film velocity, the circumferential film velocity, and the film thickness were measured at various circumferential and axial positions downstream of the film thinner for each set of flow conditions. Because of the symmetry with respect to the vertical diameter, it was not necessary to make measurements at every circumferential position. A more detailed description of the experimental equipment and procedures is

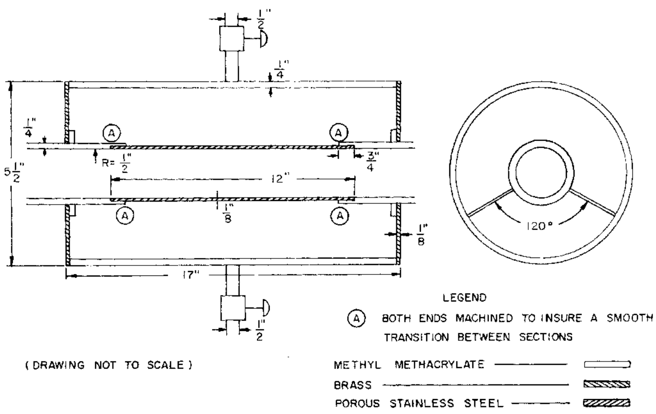

Fig. 2. Sketch of film thinner.

available from the authors upon request.

DATA ANALYSIS

The film thickness and axial velocity data were used to find the nonequilibrium values of the circumferential averaged interchange in each of two sections. The sections consisted of an 8 in. length of pipe, one section located between probes 1 and 2 and the other section located between probes 2 and 3. Because the sections consisted of the whole pipe cross section, the circumferential variation in interchange that was obtained by this analysis was value averaged over the whole pipe circumference. The averaged value of interchange in these sections was calculated from the difference between the convected film flow out of the section and the convected film flow into the section.

The mathematical model was developed by making an overall material balance on the film in the pipe section shown in Figure 3. This equation has the form

$$\overline{I}_i{}' \, \Delta z + \rho_L \int_0^{2\pi} \left[(v_z'\delta')_i - (v_z'\delta')_0 \right] \left[\frac{R - \delta'/2}{12} \right] \quad d\theta = 0 \tag{1}$$

The following assumptions were then made:

1. The film thickness was small compared with the pipe diameter. This was an excellent assumption, since the largest film thickness was about 0.03 in. compared with a pipe diameter of 1 in. That is, $(R - \delta'/2) = R$.

2. The correlation coefficient between the instantaneous axial velocity and the instantaneous film thickness was zero. In fully developed annular flow, this coefficient is definitely greater than zero, the film thickness being large at the same time that the axial velocity is large. In the present experiments, the instantaneous film thickness did not fluctuate as much as it did in fully developed annular flow, and this correlation coefficient was nearly zero. The result of this assumption is that the time average of a product is equal to the product of the time averages. That is, time average of $(v_z'\delta') = v_z\delta$.

3. The axial velocity used in the analysis was the average velocity measured for the particular section. This was also a good assumption because the measured average velocities for the two sections were close in value to one another. That is, $(v_z\delta)_i - (v_z\delta)_0 = \overline{v}_z(\delta_i - \delta_0)$.

By applying the above three assumptions and by noting the symmetry with respect to the vertical diameter, Equation (1) in terms of the time average variables became

$$\overline{I}_i = \frac{\rho_L R}{6\Delta z} \int_0^{\pi} \overline{v}_z(\delta_0 - \delta_i) d\theta \tag{2}$$

This integral was numerically evaluated by using the trapezoidal formula. Experimental measurements were made at every 20 deg. from the top to the bottom of the pipe.

The two values of interchange calculated from the above model were not the equilibrium value. After the film was thinned, the concentration of drops in the gas core decreased owing to the cessation of drop generation at the gas-liquid interface. The following model relates these two nonequilibrium values of interchange to the equilibrium value.

The starting point of this model was to make an assumption concerning the relationship between interchange and entrainment. It was assumed that the interchange was directly proportional to the entrainment; that is, $I_i = kE$.

The relationship between interchange and the axial distance downstream of the film thinner was developed by making a material balance on the entrained drops flowing in the gas core. The differential volume element is shown in Figure 4. The resulting differential equation in terms of time averaged variables has the form

$$-dE/dz - I_i = 0 \tag{3}$$

By using the proposed relationship between interchange and entrainment, that is, $I_i = kE$, and by solving the differential equation, the relationship between interchange and axial position was determined to be

$$I_i = kE(0)e^{-kz} \tag{4}$$

The boundary condition used in the solution was that at $z = 0$ the interchange was equal to k times the equilibrium entrainment, $E(0)$.

During the experimental measurements, it was observed that most of the liquid was removed in the first 3 in. of the film thinner. After this 3-in. section, the film was probably thin enough such that most drop generation had ceased. It was, therefore, decided that this axial position, 3 in. downstream from the start of the film thinner, would be used as the start of the z coordinate system.

The values of interchange calculated for each control volume were not point values for a specific axial position but were average values for the entire length of each control volume. In order to determine the value of k from the experimental data, it was necessary to obtain an expression for the average value of I_i between two axial positions from Equation (4) (that is, $\overline{I}_i$ from z_2 to z_1). This expression is

$$\overline{I}_i = \frac{E(0)}{z_2 - z_1} [e^{-kz_1} - e^{-kz_2}] \tag{5}$$

The above equation contains only one unknown, since values of $E(0)$ were available from previous experimental measurements. Two values of $\overline{I}_i$ were calculated from the experimental data, one for the control volume between z_1 and z_2 and the other for the control volume between z_2 and z_3. Since there were two equations, one for each value of $\overline{I}_i$, and one unknown k, regression was used to find the

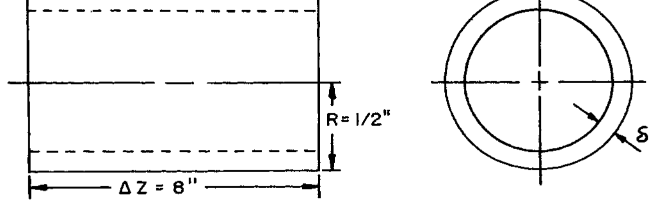

Fig. 3. Volume element for Equation (1).

best value of k. Linear regression could not be used, since Equation (5) is nonlinear with respect to k.

The best value of k was numerically determined by minimizing the sum of the squared errors. This error is defined as the difference between the experimental values and the values that were calculated from the proposed model.

The experimental data on entrainment were from the dissertation of Swanson (*11*). Swanson used a probe to measure the mass velocity of entrained drops at various positions in the gas core. These mass velocities were then numerically integrated over the core cross section to obtain the total flow rate of the entrained drops. The probe used was constructed of brass, with a circular tip 0.199 in. in diameter, and was streamlined to minimize disturbance of the flow.

Data were not available for all the flow conditions that were studied in this research. At the lower liquid rate, it was necessary to interpolate to get the value of entrainment for the superficial gas Reynolds number of 60,000. This was easily accomplished, since there were experimental data available at this liquid rate for three other gas flow rates and a smooth curve could easily be drawn through the three points.

The data were then checked for consistency. The film flow rate was determined from Swanson's data on axial velocity and average film thickness. The entrained flow was obtained from the difference between the total liquid flow and the film flow. At the lower liquid rate, the calculated values and the experimental values for entrainment were within 20% of one another. At the higher liquid rate, however, the measured values of entrainment were about double the calculated values.

The discrepancy at the higher liquid flow rate was probably caused by waves splashing into the probe when it was positioned near the gas-liquid interface at the bottom of the pipe. The entrainment mass velocity data supported this supposition. The value at the bottom of the pipe was about two or three times as great as other values. Because of this possible error in measurement at the higher liquid flow rate, the value of the entrainment calculated from the total liquid flow and film flow was used in the analysis. At the lower liquid flow rate, however, the experimental values were used.

The model used to find the circumferential variation of interchange was developed by making an overall material balance on the segment of film shown in Figure 5:

$$\frac{\partial}{\partial z}[v_z'\,\delta'\,(R-\delta'/2) + \frac{\partial}{\partial \theta}[v_\theta'\,\delta'] = (R-\delta')\,G_L'/\rho_L \qquad (6)$$

The following assumptions were then made in order to simplify Equation (6):

1. The film thickness was small compared with the pipe radius. This was a good assumption as the largest value of film thickness was about 0.03 in. compared with a pipe radius of ½ in., that is, $(R' - \delta')$ and $(R - \delta'/2) = R$.

2. The correlation coefficient between the axial velocity and the film thickness was zero. This assumption has already been discussed; that is, time average of $(v_z'\delta') = v_z\delta$.

3. The correlation coefficient between the circumferential velocity and the film thickness was zero. The tracer injection experiments verified this assumption. In fully developed annular flow, the tracer stream oscillated about the injection point indicating that at certain times the circumferential velocity was in the upward direction and that at other times it was in the downward direction. This oscillation did not occur in the measurements that were made after the film had been thinned. The tracer stream flowed only downward from the injection point in the same trajectory, regardless of the instantaneous value of film thickness. The result of this assumption was that the average of the product was the product of the averages, that is, time $(v_\theta'\delta')$ average of $v_\theta\delta$.

4. The axial velocity was not a function of the axial position. The experimental data on axial velocity exhibited the following two characteristics: the two measured values were within one standard deviation of one another and there was not any trend as to whether the value for one section was larger than the value for the other section. These characteristics made the above assumption a reasonable one (that is, $\partial v_z/\partial z = 0$).

The resulting equation in terms of time average variables became

$$v_z\frac{\partial \delta}{\partial z} + \frac{v_\theta}{R}\frac{\partial \delta}{\partial \theta} + \frac{\delta}{R}\frac{\partial v_\theta}{\partial \theta} = G_i/\rho_L \qquad (7)$$

By using Equation (7), values of interchange were ob-

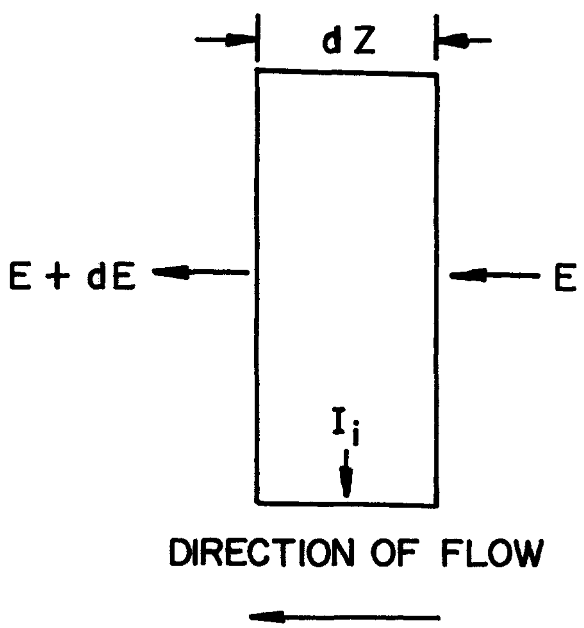

Fig. 4. Volume element for Equation (3).

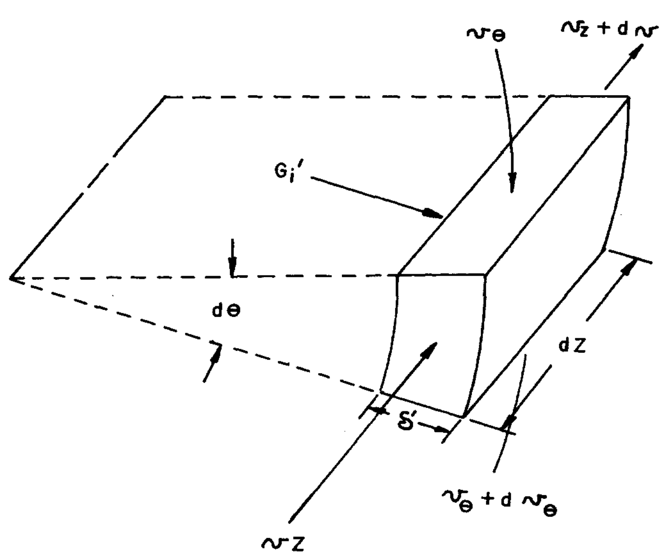

Fig. 5. Volume element for Equation (6).

TABLE 1. RESULTS OF THE CIRCUMFERENTIAL AVERAGED INTERCHANGE ANALYSIS

Case	N_{Re_L}	N_{Re_G}	$E(0)$ equilibrium entrainment (lb./sec.)	$\overline{I_i}$ z average values of nonequilibrium interchange between z_1 and z_2	between z_2 and z_3	Point estimates of k from each of two values of $\overline{I_i}$ (ft.$^{-1}$) z_1 and z_2	z_2 to z_3
A	5,000	50,000	0.0344	0.00824	0.00360	1.03	1.15
B	5,000	60,000	0.0491*	0.0140	0.00580	0.78	1.05
C	5,000	75,000	0.0750	0.0194	0.00715	0.81	1.23
D	10,000	60,000	0.0590†	0.00884	0.00485	1.08	1.00
E	10,000	75,000	0.108†	0.0148	0.00906	1.17	0.99

Case	Best pooled estimate of k from nonlinear regression (ft.$^{-1}$)	$I_i(0)$ equilibrium value of interchange (lb./sec.-ft. of pipe)	Root mean squared error $\sqrt{\sum_{L=1}^{n} \frac{(I_{i\exp} - I_{i\mathrm{pre}})^2}{n-1}}$ (lb./sec.-ft. of pipe)	Root mean squared error divided by characteristic value of I_i
A	1.12	0.0385	0.00027	0.051
B	0.94	0.0462	0.00071	0.075
C	1.10	0.0825	0.00250	0.205
D	1.06	0.0625	0.00045	0.072
E	1.11	0.120	0.00187	0.166

* Interpolated from Swanson's data (*11*).
† Calculated from Swanson's data on film thickness and axial film velocity.

tained at three axial positions downstream of the film thinner for each of ten circumferential positions. The equilibrium value of interchange was obtained by extrapolation of these three downstream values back to the zone coordinate position.

To obtain the values at the positions downstream of the film thinner, it was necessary to numerically differentiate the data to obtain values of the three required derivatives. This section will describe the techniques used to calculate these derivatives.

Before the δ vs. θ data and the v_θ vs. δ data were differentiated, the data were smoothed by using a procedure discussed by Hershey et al. (*6*). The formula chosen was a third-degree fit with five points used to obtain the coefficients in the orthogonal polynomial. The smoothing process was done four times. This procedure was suggested by the authors as one that would eliminate random error but yet leave the basic shape and character of the data unaltered.

The formula chosen for numerical differentiation was a second-order fit which used five points to obtain the coefficients in the orthogonal polynomial. This particular sloping formula was also chosen upon the suggestions of the authors as one which would give good results. This procedure was used to find both $\partial v_\theta/\partial\theta$ and $\partial\delta/\partial\theta$.

There were only three data points available with which to find the value of $\partial\delta/\partial z$. This made it necessary to use a different method to obtain values of this derivative. A second-order polynomial was fit to each group of three points. The three coefficients in this polynomial were exactly determined by the three data points. Because of this exact fit, the polynomial could have had a relative maximum in the range of interest. This would not be physically realistic, but it only occurred in one case. For various randomly selected polynomials, the fitted curve appeared to be a reasonable representation of the data. The required derivative at each of the axial positions was easily obtained from this second-order polynomial.

It was necessary to extrapolate the three numerical values of interchange to obtain the equilibrium value. This extrapolation was done graphically by extending a smooth curve through the three points when the interchange vs. axial position data were plotted on arithmetic scales, or on semilog scales (interchange on the log scale and distance on the arithmetic scale). The same results were obtained from each coordinate system.

Because of the lack of accuracy on v_θ data near the bottom of the pipe, values of interchange could be determined only for circumferential positions 0 to 140 deg. from the top of the pipe. The measured values near the bottom of the pipe were not accurate because of the complex fluid mechanics that occur at the bottom. The liquid film near the bottom does not really have a circumferential flow component. It probably acts as a stirred tank with respect to the circumferential flow. This effect could be caused by the two circumferential flows (one from the clockwise direction and one from the counterclockwise direction) meeting at the bottom of the pipe.

The value of interchange at 170 deg. was determined from the eight values at the top of the pipe along with the average value for the entire pipe circumference. Since the film thickness was small compared with the pipe radius, the relationship between the point values of interchange and the circumferential averaged value could be expressed by Equation (8):

$$\int_0^{2\pi} [G_i(0)]\,[R/12]\;d\theta = I_i(0) \tag{8}$$

Integration of the above equation was done with the trapezoidal formula by using an average value of $G_i(0)$ associated with 170 deg. for the values of $G_i(0)$ at 160 and 180 deg. Since the value of $I_i(0)$ had previously been determined, the value of $G_i(0)$ at 170 deg. could be calculated.

RESULTS

The results of the calculations for the circumferential averaged values of interchange are shown in Table 1 for the five different combinations of liquid and gas flow rates.

TABLE 2. DEPENDENCE OF k ON GAS AND LIQUID FLOW RATES FOR THE PRESENT INVESTIGATION AND EARLIER INVESTIGATIONS

	Pipe size and orientation	Liquid flow rate dependence	Gas flow rate dependence
Present investigation	1 in., hor.	0	0
Alexander and Coldren (*1*)	1.86 in., hor.	Only one rate was used	$W_G^{0.17}$
Anderson et al. (*3*)	1 in., hor.	0	$W_G^{-1.72}$
Quandt (*9*)	¼ × 3 in. × section hor.	$W_L^{-0.76}$	$W_G^{-0.3}$
Cousins et al. (*5*)	⅜ in., ver.	0	$W_G^{-0.2}$
Hoogendoorn and Welling (*7*)	2.6 in., hor.	0	0
Paleev and Filippovich (*8*)	4 × 40 mm. × section, hor.	0	$W_G^{-0.25}$

The following entries are in the table:

1. Equilibrium entrainment.
2. Experimental values of $\overline{I}_i$ for the two control volumes, between z_1 and z_2 and between z_2 and z_3.
3. Point estimates of k from the two experimental values of $\overline{I}_i$.
4. Best pooled estimate of k from the nonlinear regression.
5. Equilibrium value of interchange which was calculated from the best pooled estimate of k and the equilibrium value of entrainment.
6. Root mean squared error from the nonlinear regression analysis.
7. Normalized root mean squared error.

The root mean squared error from the nonlinear regression analysis is an estimate of quality of fit that the model exhibited. If this root mean squared error is zero, the model fits the data exactly, while a large mean squared error indicates a poor fit of the data. The magnitude of the root mean squared error alone can not be used to determine the quality of fit, but rather it must be examined in conjunction with the average measured values. In the last column of Table 1, the normalized values of the root mean squared error are shown. The normalization factor used in calculating these values was the average of the two measured values of interchange.

The approximate error in the measured values of interchange is 10%. This was calculated from the estimated errors in the axial velocity and film thickness measurements. The error in the film thickness data was estimated to be negligible, but the average error in the axial velocity measurements was estimated to be approximately 10%. Since the interchange is a linear function of the axial velocities, the estimated error in it would also be approximately 10%.

Three out of the five normalized root mean squared errors are smaller than 10%. Of the other two, the greatest value is 20.5%. These results seem to verify the assumption of proportionality between interchange and entrainment. It was necessary to verify this assumption, since it was the basis of the mathematical model which was used to extrapolate the experimental data.

By using the empirical correlations presented by Alexander and Coldren (*1*), Anderson et al. (*3*), Cousins et al. (*5*), Hoogendoorn and Welling (*7*), and Paleev and Filippovich (*8*), values of interchange were predicted for the present flow conditions. The present values are larger than the predicted values in all cases but one, the values predicted by the Cousins et al. correlation being larger than the present experimental values. This is probably caused by the effect of the pipe size. They used a ⅜ in. I.D. pipe, while the present values were for a 1 in. I.D. pipe. Except for the values from the Paleev and Filippovich correlation, all the other values are within a half order of magnitude to the present data.

There is not any discernible relationship between k for either the liquid or gas flow rate. The five k's are close in value to one another, the percentage difference between the largest and smallest values being 19.7%. This variation would be 5.4% if case B is excluded.

The present data are only for the air-water system in a 1 in. I.D. pipeline, but for this case it appears that k is independent of the gas and liquid flow rates.

Table 2 shows the dependence of k on the liquid and gas flow rates for this investigation and the earlier in-

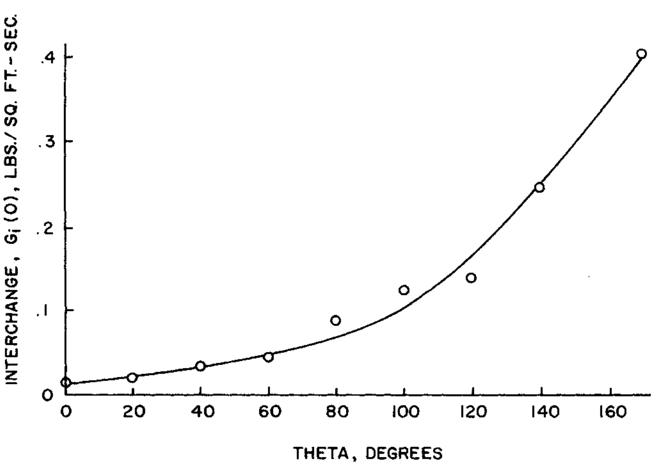

Fig. 6. Circumferential variation of interchange N_{ReL} = 5,000, N_{ReG} = 50,000.

TABLE 3. VALUES OF INTERCHANGE $G_i(0)$ AT VARIOUS FLOW CONDITIONS AS A FUNCTION OF CIRCUMFERENTIAL POSITION

Circumferential position, (deg.)	Interchange, $G_i(0)$ (lbs./sq. ft.-sec.) N_{ReL} = 5,000, N_{ReG} = 50,000	N_{ReL} = 5,000, N_{ReG} = 60,000	N_{ReL} = 5,000, N_{ReG} = 75,000	N_{ReL} = 10,000, N_{ReG} = 60,000	N_{ReL} = 10,000, N_{ReG} = 75,000
0	0.0175	0.0522	0.0835	0.0414	0.0767
20	0.0230	0.0661	0.102	0.0455	0.0890
40	0.0348	0.0832	0.125	0.0630	0.105
60	0.0477	0.0998	0.161	0.107	0.152
80	0.0890	0.154	0.183	0.162	0.0900
100	0.132	0.189	0.187	0.192	0.163
120	0.131	0.181	0.320	0.228	0.200
140	0.247	0.188	0.360	0.375	0.480
170	0.403	0.394	0.870	0.642	1.83

vestigations of interchange. The empirical correlations were rearranged into a form such that the dependence of k upon the liquid and gas flow rates could be determined. Two assumptions were made in these derivations: the cross-sectional area available for gas flow and the gas density are essentially independent of either the gas or liquid flow rates. In the present studies, both of these assumptions were excellent, the variation in both the cross-sectional area and gas density being less than 5%. It appears that this independence of k with gas and liquid flow rates is reasonable, since most of the previous experimental data also have this characteristic.

The mean residence length of the entrained liquid in the gas core is equal to the reciprocal of k. This can be shown by the following arguments: in $1/k$ ft. of pipe, the entire entrainment can be replaced by interchange, since $I_i(0) = kE(0)$. This mean residence length is not the average distance that an entrained droplet would travel in the gas core, but rather it is the average distance that a mass of liquid would travel in the gas core. In order to obtain the droplet mean residence length from the present data, it would be necessary to have information on both the size distribution of the entrained droplets and the size distribution of the droplets impinging on the film.

The five mean residence lengths calculated from the data are all about 1 ft. This value appears to be consistent with Russell's results (*10*). The residence length of a droplet that is generated at the bottom of the pipe and which impinges on the film at top of the pipe would probably be greater than the mean residence length. This was the result obtained by Russell. For a steady state injection of salt at the bottom of the pipe, he measured the salt concentration in the film at the top of the pipe at several axial positions downstream from the injection point. At the measurement position of 2-1/3 ft. downstream, there was not any salt in the film, but at a 4½ ft. measurement point, there was salt in the film. On the basis of these measurements, it appears that the residence length for a droplet which is generated at the bottom and impinges at the top of the pipe is about three and one half times greater than the mean value.

Figure 6 shows the circumferential variation of interchange for the case of $N_{Re_L} = 5{,}000$ and $N_{Re_G} = 50{,}000$. The experimental values for all cases are presented in tabular form in Table 3.

There have not been any previous experimental data on the circumferential variation of interchange to the film from the core. Indeed, it is felt that the present data are only semiquantitative, owing to the measurement errors in the circumferential film velocity. However, the trends expressed by the data are probably correct. The values of interchange in the top third of the pipe are all the same order of magnitude. Values in the rest of the pipe exhibit a sharp increase with angular position, there being an order of magnitude difference between the values of interchange at the bottom and top of the pipe.

The present experiments were done in a 1 in. I.D. pipe. A proposed model to extrapolate the circumferential averaged values of interchange to other pipe sizes is

$$kR = \text{constant} \tag{9}$$

The development of this method is not rigorous but instead is based on an argument concerning the mass mean residence length. As a first-order approximation, it could be assumed that the axial distance that a drop travels is proportional to the radial distance that the drop must travel in order to impinge on the film. For a given initial drop path, the radial distance that a drop would travel before impinging on the film would be proportional to the pipe radius. The above arguments can be expressed by the following equation:

$$\overline{L} \propto R \tag{10}$$

If the expression for $\overline{L}$ in terms of k, that is, $\overline{L} = 1/k$, is substituted into Equation (10), Equation (9) is obtained.

Hoogendoorn and Welling (*7*) measured interchange in two different horizontal pipes of different sizes; one was a 2 in. I.D. and the other a 6 in. I.D. The above expression adequately correlated their results for the two different size pipes.

If there are no entrainment data available, the best correlation with which to predict a value is that of Wicks and Dukler (*13*).

CONCLUSIONS

1. An experimental technique has been developed to measure interchange in annular flow. The experimental apparatus consists of a device to reduce the film thickness and probes which are used to measure the film thickness, axial film velocity, and circumferential film velocity. The film thickness is reduced to a small enough value so that interchange from the film to the core ceases. Interchange from the core to the film is determined by measuring the rate of film buildup downstream of the film thinner.

The circumferential averaged values of interchange were determined as well as the circumferential variation of interchange to the film from the core.

2. The circumferential averaged value of interchange is directly proportional to the flow rate of entrained drops. This proportionality constant was found to be essentially independent of the gas and liquid flow rates. For the five combinations of gas and liquid flow rates which were studied, the constant ranged in value from 0.94 to 1.12 ft.$^{-1}$.

3. Interchange was found to be a monotonically increasing function of circumferential position (0 deg. is at the top of the pipe). The ratio of the value at the bottom of the pipe to the value at the top is about 10.

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NOTATION

No superscript on a time dependent variable denotes the time average of that variable.

E = flow rate of entrained drops, lb./sec.
$E(0)$ = equilibrium value of entrainment, lb./sec.
G_i = interchange to the film from the core as a function of circumferential position, lb./(sq.ft.)(sec.)
$G_i(0)$ = equilibrium value of interchange to the film as a function of circumferential position, lb./(sq.ft.)(sec.)
I_i = interchange to the film from the core averaged over entire pipe circumference, lb./(ft.)(sec.)
$I_i(0)$ = equilibrium value of interchange to the film which has been averaged over the entire pipe circumference, lb./(ft.)(sec.)
k = proportionality constant between interchange and entrainment, ft.$^{-1}$
n = number of data points in the nonlinear regression analysis

N_{ReG} = superficial gas Reynolds number
N_{ReL} = superficial liquid Reynolds number
R = pipe radius, in.
v_z = axial velocity of the film, ft./sec.
v_θ = circumferential velocity of the film, ft./sec.
z = coordinate direction parallel to the pipe axis positive direction is downstream, in.
δ = film thickness, in.
θ = angular direction, 0 deg. at the top of the pipe, 180 deg. at the bottom of the pipe, deg. or rad.
ρ_L = liquid density, lb./cu.ft.

Subscripts and Superscripts

exp = experimentally determined value of a variable
pre = value of a variable calculated from a proposed model
1 = coordinate position
2 = coordinate position where 2 is greater than 1
bar = average value of a variable with respect to a coordinate direction
tick = instantaneous value of a variable

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Continuous Filter Cake Washing Performance

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A previous report (1) has presented the derivation of new filter cake washing equations. This report presents the procedure for their application and contains an example of application to a specific batch washing system. It also presents the adaption of the equations to permit their use in predicting the performance of a continuous rotary-drum filter.

Filtration accomplishes the removal of undissolved solid material from a liquid by passing slurries through a filter medium in either a continuous or discontinuous operation.

In many slurries, the filtrate contains dissolved material, called the *solute.*

If the filtrate retained in the filter cake is not removed before further operations, such as drying, this dissolved material will remain in the cake as the liquid is removed. Washing is used for the recovery or reduction of solute content in the cake. In the study of continuous washing processes, the mechanism of cake formation as well as cake washing performance must be well known.

Choudhury and Dahlstrom (*2*) employed Rhodes washing equation (*4*) to describe washing performance in a continuous rotary-drum filter operation. From commonly accepted filtration theory, the wash time is correlated as a function of wash ratio with parameters of cake formation time. (Note: wash ratio R is defined as

$$R = \frac{\text{volume of wash liquor}}{\text{volume of original liquor in cake}}.)$$

Their correlation of recovery ratio as a function of wash ratio appears to be satisfactory. (Note: recovery ratio = weight fraction of solute remaining in cake after washing.)

A new filter cake washing performance equation has been derived (*1*). It assumes plug flow of the washing liquor through pore channels in the filter cake. The filtrate to be recovered is in the cake as a stagnant film on the